Meeting the FCC's Mobile Location Indoor Accuracy Requirements for 911 Calls

White Paper

Prepared by: AdGen Telecom Group, Inc.

5/8/2014

Table of Contents

Executive Summary

Mobile positioning refers to the geographic coordinates of a mobile device in a wireless network or wireless local area network (WLAN). The Federal Communications Commission (FCC) has mandated that all wireless service providers deliver the position of an enhanced 911 (E911) caller to public safety answering points (PSAPs). The FCC mandate on positioning in wireless networks is to address a fundamental public safety issue in that a large proportion of the E911 calls originate from mobile phones where the caller is unable to provide an accurate location either outdoors or indoors in urban, suburban or rural areas. In addition to emergency services, mobile positioning information can also be used for a number of other services, including monitoring and tracking for purposes of security, location sensitive billing, fraud protection, asset tracking, fleet management, intelligent transportation systems, mobile advertising and mobile yellow pages. In the future, wireless networks could also use positioning information for intelligent resource management.

The FCC's Notice of Proposed Rule Making (NPRM), released on March 28, 2014, seeks to expand existing E911 location requirements to include indoor location accuracy for mobile E911 calls. The proposed requirements are as follows:

- Wireless carriers are required to provide "horizontal" indoor location accuracy that locates callers within 50 meters for 67 percent of calls within two years and 80 percent of calls within five years.
- Wireless carriers are required to provide "vertical" (i.e., floor level) indoor accuracy that locates callers within three meters for 67 percent of calls within three years and 80 percent of calls within five years.

This White Paper presents an overview of positioning technologies, their challenges and limitations, and associated position accuracies, and then illustrates the relationship between the FCC's proposed indoor accuracy standards and existing technologies. There are several mobile positioning technologies based on different wireless measurements, processing and algorithms. However, meeting these FCC objectives in different morphologies (by the positioning techniques) is a challenge due to the uncertain measurements caused by the difficulties intrinsic to the wireless environment, such as non-line of sight in dense urban and indoor scenarios, fading, low Signal to Interference-Noise Ratios (SINRs), multi-user interference, multipath conditions, etc. Each one of the positioning techniques has different fundamental limitations, which results in different accuracies.

In summary, TDOA network-based technology is a technically feasible and economically reasonable solution for indoor location accuracy. The estimated accuracy of the TDOA network-based technology (using nominal network performance values) is improved when both the mobile power and the number of the TDOA receivers are increased. The findings of this paper demonstrate that a modest increase in handset power and number of TDOA receivers would substantially improve the existing accuracy of the location of the mobile device in urban and/or dense urban areas. Moreover, this paper reveals that using some nominal network performance parameters (i.e., SINR = 10 dB[1]), available TDOA network-based technology can improve the accuracy of the mobile location to 31 to 76 meters in 67 percent of the calls. It is conceivable that the accuracy of the TDOA network-based technology can be further improved should the network operate at: (a) higher mobile power and (b) with an increased number of TDOA receivers.

[1] - The SINR includes the processing gain

I. Background

The FCC adopted the rules for the Commercial Mobile Radio Service (CMRS) operators to provide E911 services in two phases in 1996. The first phase (Phase I) was to be implemented by April 1998 and required the CMRS carriers to provide the caller telephone number (i.e., Caller Identification) and the location of the base station (i.e., latitude and longitude of the origination cell tower) to the PSAP for all 911 calls originated by mobile subscribers. The second phase (Phase II) requires CMRS carriers to provide the latitude and longitude of the 911 calls to the PSAPs within a certain accuracy and reliability for outdoor 911 calls, where the location accuracy depends on the technology used by the CMRS operators. The technologies commonly used and available to CMRS operators are:

(a) Network based technologies with an accuracy of 100 meters for 67 percent of calls, and 300 meters for 90 percent of calls, and
(b) Handset based technologies (Assisted GPS), with 50 meters for 67 percent of calls, and 150 meters for 90 percent of calls.

The current rules do not include location accuracy requirements for indoor 911 calls. To determine the feasibility of providing indoor accuracy for mobile 911 calls, the FCC's Communications Security, Reliability and Interoperability Council (CSRIC) III released a report in June of 2012 about its objective to conduct a test bed in all different types of morphologies, to provide insight into what technologies are available for use in providing the indoor location of a mobile subscriber's 911 calls. San Francisco was selected for the test bed and the following technologies were used for the indoor location accuracy:

(a) Assisted Global Positioning System (AGPS)/ Advanced Forward Link Trilateration (AFLT),
(b) RF fingerprinting, and
(c) In building network beacon.

The results of the test bed were issued in 2013 and "show[ed] significant promise with respect to high yield, relatively high confidence factors and reliability," and "the ability to achieve improved search rings in the horizontal dimension (often identifying the target building, or those immediately adjacent)."

Following the San Francisco test, a hybrid technology (AGPS)/Uplink-Time Difference of Arrival - UTDOA) for indoor locations was performed. The test results supported the earlier finding of the CSRIC published report with accuracy levels similar to the horizontal accuracy stated in the FCC's Third Further Notice Of Proposed Rule Making (NPRM), DA 14-420, released March 28, 2014 [1].

II. Mobile Location Technologies

This White Paper presents an overview of various mobile location technologies, their challenges and limitations, and associated position accuracies, and illustrates the relationship between the FCC's proposed indoor accuracy standards and existing technologies. Today, there are several mobile location technologies in wireless networks that use wireless network information and satellites. The mobile positioning technologies fall into two main categories: *"Handset-based"* and *"Network-based."*

A. Handset-Based Technology

There are several handset-based technologies available for locating mobile devices. Global positioning system (GPS) is the most commonly used handset based technology. Other handset-based technologies include Beacon Technology, Enhanced - Observed Time Difference (EOTD), Observed Time Difference Of Arrival (OTDOA) and Advanced Forward Link Trilateration (AFLT), which are briefly described below.

1. Global Positioning System (GPS)

In handset-based location technology the mobile device determines its location from signals received from satellites in the GPS. The mobile device receives and measures the signal parameters from four (4) GPS satellites, with a clear line-of-sight (LoS) to GPS satellites. The mobile handset measures the time it takes for the received signal from each satellite to reach the mobile device. The location provided by the GPS system has a high degree of accuracy. The nominal accuracy of the GPS system is approximately 10 meters.

The primary disadvantage of GPS measurement is that it is highly dependent on the LoS; therefore, in areas where LoS is obstructed by buildings, GPS cannot be used to provide an accurate location. The additional disadvantage of using a GPS receiver is that it increases battery consumption as well as the cost of the handset. Moreover, GPS has: (a) a long acquisition time of at least 30 seconds [2], [3] and (b) inaccurate positioning. This is highly undesirable for emergency situations occurring while turning the mobile device on. In comparison, a network-based technology can provide the mobile location in less than ten (10) seconds [4], which is much less than the GPS acquisition time. Therefore, it may not be wise to embrace GPS fully as the sole or primary E911 location technology under the FCC's proposed indoor location standards.

2. Beacon Technology

Beacon technology is a new network and handset-based technology developed and utilized by NextNav called Metropolitan Beacon System (MBS). According to NextNav literature [5], the MBS concept shares many operating principles with GPS, but because the NextNav beacons are installed terrestrially instead of in space, they transmit sufficient signal strength for precise and reliable indoor reception and in urban areas with no LoS.

Moreover (again according to NextNav) the MBS is:

- Deployed much like a cellular network, operating on licensed 900 MHz band spectrum, to provide consistent indoor positioning to every building within a covered metropolitan area;
- Deploying a nationwide positioning system based on a fully managed network, based on owned assets; and
- An air interface-agnostic system that allows for evolution of location technology independent of air interface (2G, 3G, 4G, etc.).

3. Enhanced - Observed Time Difference (EOTD)

EOTD operates only on GSM and GPRS networks [6]. The location of the mobile, using EOTD is determined by:

(a) mobile measure timing of signals from nearby cell sites,,
(b) measurement reported back to the network, and
(c) A network element (SMLC) uses the timing measurements to compute the location of the mobile handset.

ETDOA technology requires handsets that are compatible with it. This method includes new technology in the handset to assist in locating the unit in a network.

4. Observed Time Difference Of Arrival (OTDOA)

OTDOA is a handset-based positioning technology that is introduced in 3rd Generation Partnership Project (3GPP), Released 9 for Long Term Evolution (LTE). The location of the mobile is determined by making timing offset measurements between the LTE serving eNodeB and neighbor eNodeB, called received signal time difference (RSTD) measurements.

5. Advanced Forward Link Trilateration (AFLT)

Advanced Forward Link Trilateration (AFLT) was developed by Qualcomm in2000. AFLT in conjunction with GPS provides the location of the mobile device. According to Qualcomm [18], AFLT works by:

- A mobile measuring of the phase of CDMA Pilot signals that it receives;
- A location server associates each Pilot measurement to a nearby base station;
- A database is used to maintain the location of each base station in the network; and
- Trilateration is used to turn the base station positions and Pilot measurements in to a position solution similar to GPS.

AFLT is presently supported only on the CDMA network.

B. Network-Based Technologies

Network based location technologies determine the position of a mobile device by measuring its received signal parameters at the receivers used for positioning. In these technologies, the receivers (for the positioning system) measure the signals transmitted from a mobile device and relay the measurements to a central server for further processing, which provides an estimate of the mobile device location. A significant advantage of network-based technologies is that the mobile device is not involved in the positioning process; therefore, these technologies do not require modifications to existing handsets.

There are two phases for network-based technologies: 1) the receivers (for the positioning system) have to measure signal parameters (such as energy, time or angle of arrival) of the received mobile device signal; and 2) the measured signal parameters are analyzed at the positioning server to provide the final estimate for the location of the mobile device. Figure-1 illustrates the two phases for a three base stations scenario.

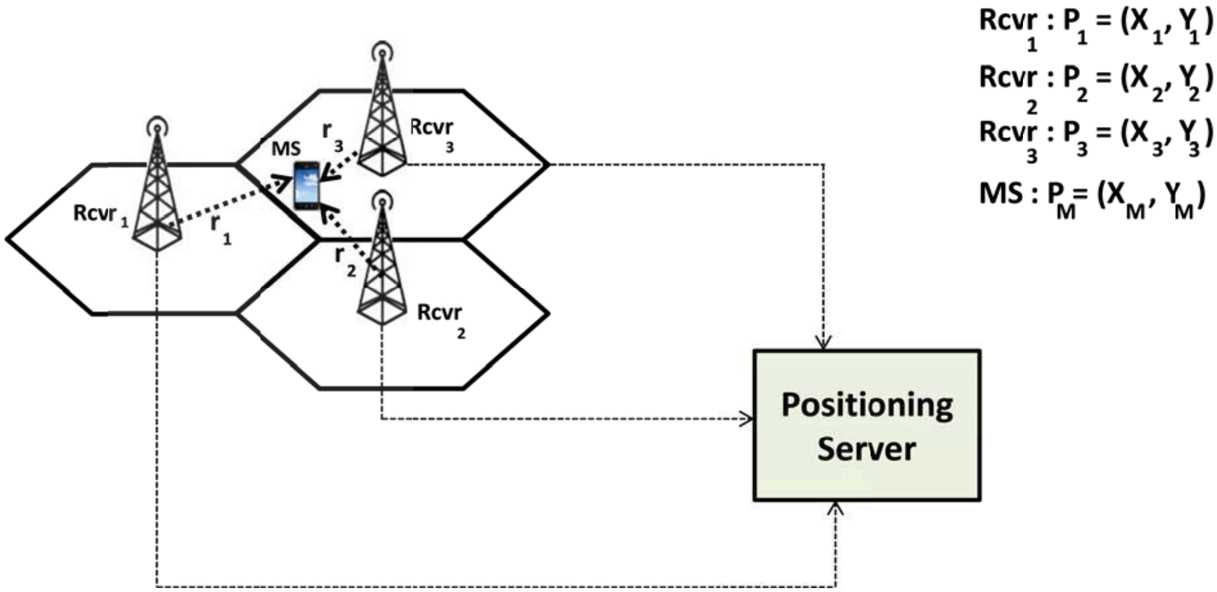


Figure-1: Network based positioning for three base stations

The location of the receiver "i" (for positioning system) and the location of the mobile device are denoted by $P_i = (X_i, Y_i)$ and $P_M = (X_M, Y_M)$, respectively. Also, the distance between the mobile device and the receiver "i" is denoted by "r_i" (shown in Figure-1), where:

$$\mathbf{r_i = |P_M - P_i| = \sqrt{(X_M - X_i)^2 + (Y_M - Y_i)^2}} \quad \textbf{(Equation-1).}$$

This assumes the positioning system knows the distances from the mobile device to three positioning receivers ($r_1 = |P_M - P_1|$, $r_2 = |P_M - P_2|$, $r_3 = |P_M - P_3|$) as shown in Figure-1. Knowing the position of the three positioning receivers ($P_1 = (X_1, Y_1)$, $P_2 = (X_2, Y_2)$, $P_3 = (X_3, Y_3)$), the location of the mobile device $P_M = (X_M, Y_M)$ can be accurately obtained as shown in Figure-2.

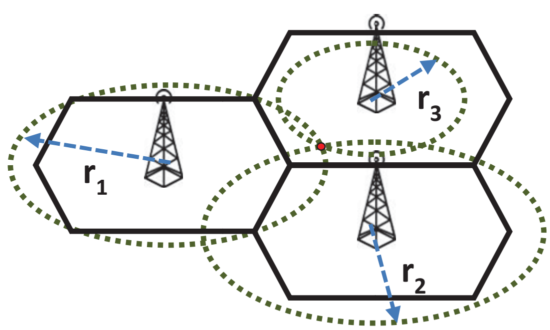


Figure-2: Obtaining the location of the mobile device knowing its distance from three positioning receivers.

In practice, the positioning system does not know the accurate distance of the mobile device from the positioning receiver "i" (r_i), rather, it indirectly measures the distance ($\hat{r}_i$), and $\hat{r}_i = r_i + e_i$ where e_i is the measurement error. The measurement error of the distance from the positioning receiver ($\Delta_i = |e_i|$) is affected by a number of propagation anomalies (e.g., low SNIR, fading, etc.), some system parameters and morphology. Due to this measurement error, the mobile station is not necessarily located on a circle of radius "r_i", instead it lies inside a ring around the base station "i" with inner and outer radiuses of "$r_i \mp \Delta_i$", as shown in Figure-3.

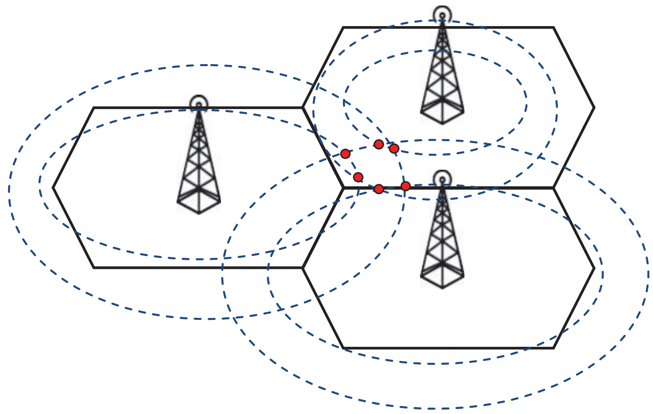


Figure-3: Due to the errors, the location of the mobile device lies in the area within the red dots instead of a single point.

All the mobile positioning technologies make measurements of few parameters, which are functions of the mobile device position (P_M). In the above example, the measurements are the distances of the mobile device from three positioning receivers ($r_i = f(P_M, P_i)$). In wireless networks, measurements are corrupted by some error (as previously explained); hence, what the system actually measures is a function of the mobile device location and some degree of error: $y = \hat{r}_i = f(P_M, P_i) + e$. This error "$e$" is modeled as a Gaussian random variable: $p_E(e) \sim N(\mu, \sigma)$.

An estimation method can be used to determine the position of the mobile device (P_M) from the noisy measurements "y"; the Cramer-Rao lower bound provides the accuracy of the positioning for any unbiased estimator [7], [8]. It is known that the maximum likelihood estimator asymptotically achieves the Cramer-Rao bound [9]. The estimation method finds the position ($\hat{P}_M$) that minimizes a given norm ($V(P_M)$) of the difference of actual measurements and the measurement model [10]:

$$\hat{\boldsymbol{P}}_{\boldsymbol{M}} = \boldsymbol{arg\,min}_{\boldsymbol{P}_{\boldsymbol{M}}} \boldsymbol{V}(\boldsymbol{P}_{\boldsymbol{M}}) = \boldsymbol{arg\,min}_{\boldsymbol{P}_{\boldsymbol{M}}} \|\boldsymbol{y} - \boldsymbol{f}(\boldsymbol{P}_{\boldsymbol{M}})\| \quad \textbf{(Equation-2).}$$

There are several common estimation methods as options for mobile positioning including: maximum likelihood (ML), minimum mean squared error (MMSE), maximum a posteriori (MAP), weighted nonlinear least squares (WNLS), Gaussian maximum likelihood (GML), etc. These estimators are summarized in Table-1 [10]:

Optimization criteria $V(P_M)$ for estimating position P_M from uncertain measurements: $y = f(P_M) + e$		
Maximum Likelihood	*ML*	$\boldsymbol{V^{ML}(P_M) = \log p_E(y - f(P_M))}$
Minimum Least Squared Error	*MLSE*	$\boldsymbol{V^{MLSE}(P_M) = \|y - f(P_M)\|^2 = (y - f(P_M))^T (y - f(P_M))}$
Weighted Nonlinear Least Squared Error	*WNLSE*	$\boldsymbol{V^{WNLSE}(P_M) = (y - f(P_M))^T R_t^{-1}(P_M)(y - f(P_M))}$
Gaussian Maximum Likelihood	*GML*	$\boldsymbol{V^{GML}(P_M) = (y - f(P_M))^T R_t^{-1}(P_M)(y - f(P_M)) + \log\det R_t(P_M)}$

Table-1: Several common estimation methods that can be used in mobile positioning.

There are several network-based positioning technologies with different measured parameters and different data processing techniques:

1. Cell Identification (CID),
2. Received-Signal-Strength (RSS),
3. Time-of-Arrival (TOA),
4. Time-Difference-of-Arrival (TDOA),
5. Angle-of-Arrival (AOA),

6. Radio Frequency (RF) Fingerprinting,
7. Advanced Forward Link Trilateration (AFLT), and
8. Beacon Technology.

Each technology has its own merits and drawbacks, depending upon the cost and complexity constraints. The next sections will review these network- based location methods.

1. Cell Identification (CID)

Cell identification (CID) is the process of using a network's knowledge of the mobile device, within the controlling cell site/sector, and communicating the cell/sector information. CID provides a very rough estimate of the location of a mobile device. Cells (base stations) provide coverage for wireless networks and each cell is divided into three sectors. Cell size varies from less than one kilometer in dense urban areas, to roughly one to three kilometers in urban areas, and up to 20 km in suburban/rural areas. Although a simple technology, CID positioning accuracy is very low and does not meet the FCC's requirements for accuracy.

2. Received Signal Strength (RSS)

In some wireless technologies, the total path attenuation can be measured or estimated. The total path attenuation is a combination of three factors: 1) free space path-loss, 2) shadow fading and 3) fast fading. Taking the time average of total path attenuation cancels the fast fading factor, so the average total path attenuation is a function of free space path-loss and shadow fading.

The Okumura-Hata model relates the free space path-loss between the mobile device and positioning receiver "i" to the distance between them "$r_i = |P_M - P_i|$": $Path - loss = K + 10\alpha \log_{10}(r_i)$; therefore, by adding the shadow fading effect, the average total channel attenuation would be [11], [12]:

$$\boldsymbol{Avg\ channel\ attenuation = y_i = K + 10\alpha\, log_{10}(r_i) + e = K + 10\alpha\, log_{10}|P_M - P_i| + e_i} \quad \textbf{(Equation-3)}$$

where "e_i" is zero, this means the Gaussian random variable represents shadow fading with a standard deviation of 4−12 dB, depending on the morphology [10]. Here the measurement ("y_i") is the path attenuation from the mobile station to the positioning receiver "i" which is a function of the mobile device position ("P_M"). Having the path attenuation from the mobile device to at least three positioning receivers, the mobile device position ("P_M") can be estimated by using the estimation methods previously mentioned.

Shadow fading and multipath cause errors in RSS measurements that result in inaccuracy in estimating the position of the mobile device. RSS measurements depend on the path characteristics; and, the RSS based positioning algorithms are sensitive to channel parameters estimation. Because of these errors, especially in dense urban areas, the RSS mobile positioning method cannot meet the FCC's requirements and does not seem to be a very reliable method for mobile location in wireless networks.

3. Time-of-Arrival (TOA) Based Positioning

In the time-of arrival (TOA) positioning method, the one-way propagation times of the signal traveling between the mobile device and a number of the positioning receivers are measured and the location of the mobile device is obtained by combining the estimates of the TOA of the mobile device signal arriving at different positioning receivers [13], [14].

Since the wireless signal travels at the speed of light ($c = 3 \times 10^8\ {}^{m}/_{sec}$), the distance between the mobile device and the positioning receiver "i" can be measured by:

$$\mathbf{r_i = |P_M - P_i| = (t_i - t_0)c} \quad \textbf{(Equation-4)}$$

where "t_0" is the time when the mobile device transmits a signal and "t_i" is the time of arrival of the signal at the positioning receiver "i." In this positioning method, the measurement is "$y_i = {}^{1}/_{c}\,|P_M - P_i| + e_i$" at the positioning receiver "i" where "e_i" is a Gaussian random variable: $p_E(e_i) \sim N(\mu, \sigma)$. Travel time can be measured in many different ways, e.g., it is estimated in the uplink to multiple positioning receivers upon the request of the network [10]. If measurements from three or more positioning receivers are available, the position of the mobile device can be estimated using one of the estimation methods previously mentioned. TOA is depicted in Figure-4.

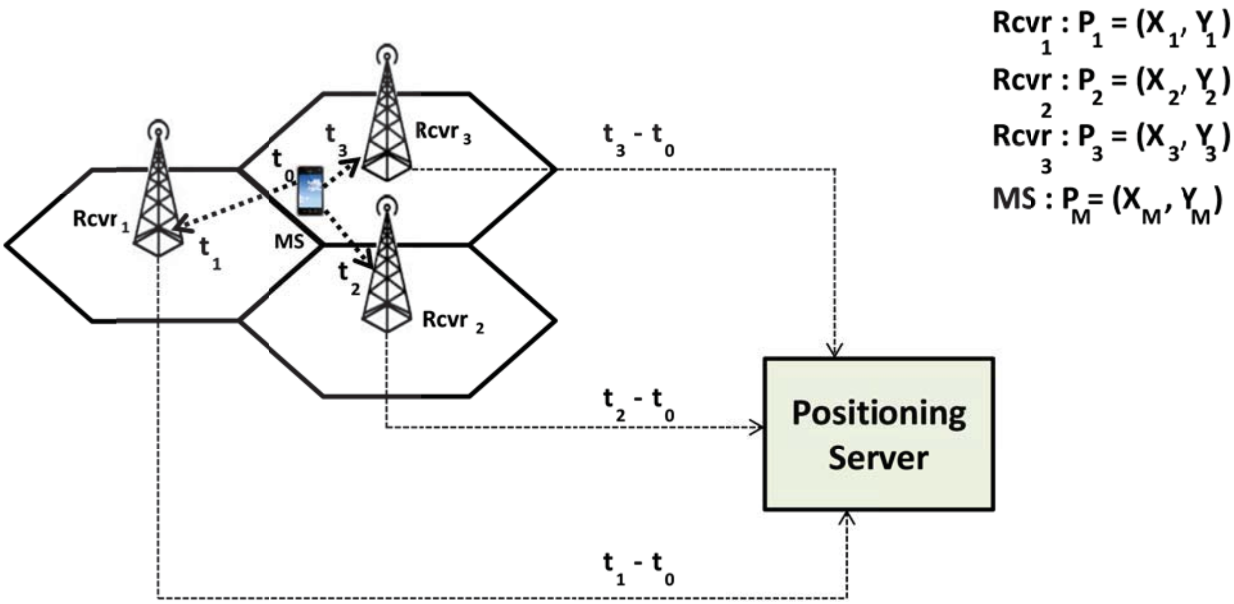


Figure-4: Time-of-Arrival (TOA) positioning mechanism

TOA requires measurements from at least three positioning receivers; consequently, location estimation cannot be performed in areas without at least three visible positioning receivers. In urban and suburban areas, where the FCC's proposed indoor location requirements are most likely to be needed, obtaining three positioning receivers should not be a problem; however, in rural areas this can be a problem for TOA. Also, the TOA positioning method relies on an accurate estimation of signal travel time and line-of-sight between the mobile device and the positioning receivers. In practice, because of lack of full synchronization and lack of line-of-sight (LoS) in TOA, the measured distance of the mobile device to the positioning receiver "i" would be $\tilde{r}_i = r_i + e_i$, where e_i is the measurement error. This sometimes leads to inaccuracy in TOA positioning.

In a fully synchronized wireless network, the signal travel time can be accurately estimated. Time synchronization in wireless networks is achievable among the positioning receivers. However, the mobile device clock is not synchronized to the positioning receiver's clock and depending on the mobile chip rate its clock drifts, which can generate an error in the location estimate of the TOA method.

Another source of error in TOA positioning also happens when there is no line-of-sight (NLoS) from the mobile device to the positioning receivers and multipath. This usually occurs in urban/dense urban and indoor scenarios. In wireless networks where the direct path from the mobile device to a positioning receiver is blocked, the signal measurement is corrupted because of NLoS error. To reduce the effect of the NLoS error, several techniques, such as geometrically constrained data processing schemes and constraint optimization approaches, have been proposed [13]. In addition, other methods are proposed [15] to enhance the accuracy of TOA and mitigate the effect of NLoS error by processing the multipath delays.

3.1 TOA Positioning Using Least Square Estimator

Least square estimator (LSE) can also be used for TOA positioning [13] and is briefly described below. Let us assume there are "N" positioning receivers at known locations: $P_1 = (X_1, Y_1), P_2 = (X_2, Y_2), \cdots, P_N = (X_N, Y_N)$. Since the TOA measurements are proportional to the distances of the mobile device and the positioning receivers ($y_1 = {}^1/_c\,\hat{r}_1,\ y_2 = {}^1/_c\,\hat{r}_2, \cdots, y_N = {}^1/_c\,\hat{r}_N$), the position of the mobile device ($P_M = (X_M, Y_M)$) can be estimated. It can be shown that the following linear equation holds:

$$\boldsymbol{HP_M = b} \qquad \textbf{(Equation-5)}$$

where:

$$H = \begin{bmatrix} X_2 - X_1 & Y_2 - Y_1 \\ X_3 - X_1 & Y_3 - Y_1 \\ \vdots & \\ X_n - X_1 & Y_N - Y_1 \end{bmatrix}, \quad P_M = \begin{bmatrix} X_M \\ Y_M \end{bmatrix}, \quad b = \frac{1}{2}\begin{bmatrix} X_2^2 + Y_2^2 - \hat{r}_2^2 + \hat{r}_1^2 \\ X_3^2 + Y_3^2 - \hat{r}_3^2 + \hat{r}_1^2 \\ \vdots \\ X_N^2 + Y_N^2 - \hat{r}_N^2 + \hat{r}_1^2 \end{bmatrix}$$

The least square estimation (LSE) for the mobile device position in TOA would be:

$$\boldsymbol{\hat{P}_M = (H^T H)^{-1} H^T b} \qquad \textbf{(Equation-6).}$$

4. Time-Difference-Of-Arrival (TDOA) Based Positioning

The TOA positioning method requires accurate synchronization between the mobile device and the positioning receivers. In a wireless network, it is a challenge to synchronize the transmission time of the mobile device signal at time "t_0" with the clock of the positioning receivers. Taking the time differences of the TOA measurements eliminates "t_0", and therefore resolves this clock issue. The TDOA measurement associated with the positioning receiver "i" is $y_i = (t_i - t_0) + (t_1 - t_0) = (t_i - t_1)$, i.e., the difference between the TOA measurements of the mobile device signal at the positioning receivers "i" $(t_i - t_0)$ and positioning receiver "1" $(t_1 - t_0)$ [2], [13], [14], [16], [17].

The measurement in TDOA is a noisy function of the mobile device position (P_M). The noisy measurement of the positioning receiver "i" is as follows:

$$\boldsymbol{y_i = \frac{1}{c}\left\{|P_M - P_i| - \frac{1}{c}|P_M - P_1| + e_i - e_1\right\}} \qquad \textbf{(Equation-7)}$$

where "e_i" and "e_1" are Gaussian random variables: $p_E(e_i)\,, p_E(e_1) \sim N(\mu, \sigma)$. Having the measurements of TDOA in at least three positioning receivers, any one of the standard estimation techniques can be used to estimate the location of the mobile device.

Some studies [10] use simulations to demonstrate that TDOA position accuracy; in NLoS (e.g., urban/dense urban) it is approximately 100 meters for 67 percent of the calls. Other studies [3] use simulations to show that TDOA position accuracy can be 50-120 meters for 67 percent of the calls. There are other studies [2] that use simulations to show that for a CDMA network, the TDOA position accuracy is approximately 125 meters for 67 percent of the calls. Therefore, the position accuracy range for TDOA for urban/dense urban is 50 to 125 meters for 67 percent of the calls.

4.1 TDOA Positioning Using Least Square Estimator (LSE)

Similar to TOA, the LSE can be used to estimate the location of the mobile device ($P_M = (X_M, Y_M)$) [13]. There are "N" positioning receivers in the TDOA positioning process located at known locations: $P_1 = (X_1, Y_1), P_2 = (X_2, Y_2), \cdots, P_N = (X_N, Y_N)$. In TDOA, the difference of the estimated distance from the mobile device to the positioning receiver "i" and estimated distance from the mobile device to the positioning receiver "1" can be obtained from Equation-8:

$$\hat{\boldsymbol{r}}_{i1} \triangleq \boldsymbol{y}_i \boldsymbol{c} = \hat{\boldsymbol{r}}_i - \hat{\boldsymbol{r}}_1 = (\boldsymbol{t}_i - \boldsymbol{t}_0)\boldsymbol{c} - (\boldsymbol{t}_1 - \boldsymbol{t}_0)\boldsymbol{c} = (\boldsymbol{t}_i - \boldsymbol{t}_1) \qquad \textbf{(Equation-8)}$$

The objective is to obtain the position of the mobile device, $P_M = (X_M, Y_M)$. The following linear equation holds:

$$\boldsymbol{HP_M} = \hat{\boldsymbol{r}}_1 \boldsymbol{c} + \boldsymbol{d} \qquad \textbf{(Equation-9)}$$

where:

$$H = \begin{bmatrix} \boldsymbol{X_2 - X_1} & \boldsymbol{Y_2 - Y_1} \\ \boldsymbol{X_3 - X_1} & \boldsymbol{Y_3 - Y_1} \\ \vdots & \\ \boldsymbol{X_n - X_1} & \boldsymbol{Y_N - Y_1} \end{bmatrix}, \quad \boldsymbol{P_M} = \begin{bmatrix} \boldsymbol{X_M} \\ \boldsymbol{Y_M} \end{bmatrix}, \quad \boldsymbol{c} = \begin{bmatrix} -\hat{\boldsymbol{r}}_{21} \\ -\hat{\boldsymbol{r}}_{31} \\ \vdots \\ -\hat{\boldsymbol{r}}_{N1} \end{bmatrix} \quad \boldsymbol{d} = \frac{\boldsymbol{1}}{\boldsymbol{2}} \begin{bmatrix} \boldsymbol{X_2^2 + Y_2^2 - \hat{r}_{21}^2} \\ \boldsymbol{X_3^2 + Y_3^2 - \hat{r}_{31}^2} \\ \vdots \\ \boldsymbol{X_N^2 + Y_N^2 - \hat{r}_{N1}^2} \end{bmatrix}$$

and: $\hat{\boldsymbol{r}}_1 = \sqrt{\left(\hat{\boldsymbol{X}}_M - \boldsymbol{X}_1\right)^2 + \left(\hat{\boldsymbol{Y}}_M - \boldsymbol{Y}_1\right)^2}$

The LSE for the mobile device position in TDOA would be:

$$\hat{\boldsymbol{P}}_M = (\boldsymbol{H}^T \boldsymbol{H})^{-1} \boldsymbol{H}^T (\hat{\boldsymbol{r}}_1 \boldsymbol{c} + \boldsymbol{d}) \qquad \textbf{(Equation-10).}$$

5. Angle of Arrival (AOA) Positioning

Angle of arrival (AOA) positioning technology involves measuring the angles of the mobile device as seen by the positioning receivers. The position of the mobile device is calculated from the intersection of a minimum of two lines connecting the mobile device and two positioning receivers, as shown in Figure-5 [13]. Knowing the location of the positioning receivers ($P_1 = (X_1, Y_1), P_2 = (X_2, Y_2)$) and the angles of the two lines connecting the mobile device and the positioning receivers (α_1, α_2), the location of the mobile device ($P_M = (X_M, Y_M)$) can be obtained.

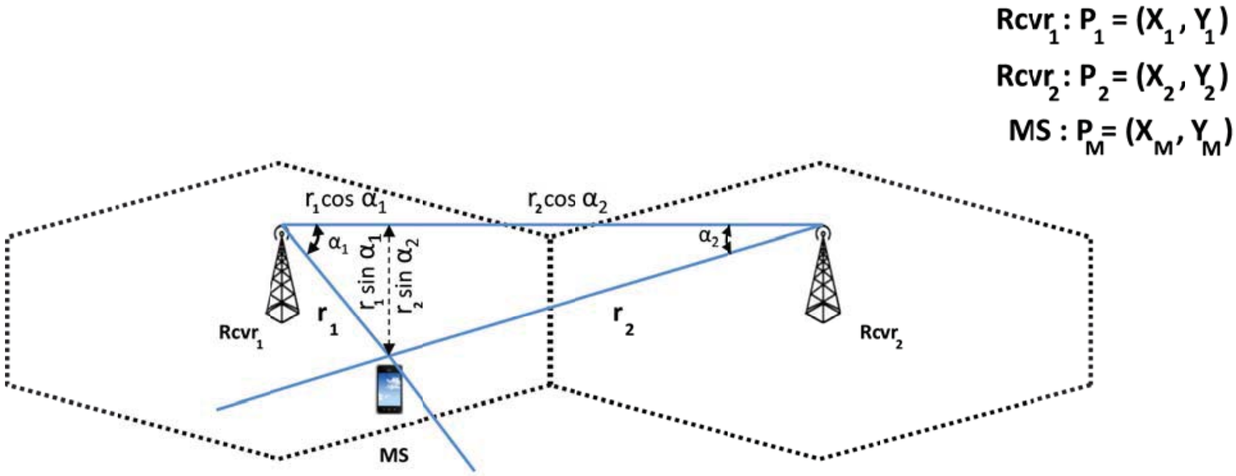


Figure-5: Angle of arrival (AOA) positioning

The AOA signals at the positioning receivers are determined by electronically steering the main lobe of an adaptive phased array antenna in the direction of the arriving mobile device signal [13]. An adaptive phased array antenna system consists of an array of sensor elements and a real-time adaptive signal processor. The system can automatically adjust the antenna's beam pattern, frequency response and other parameters to enhance location performance. If a mobile device transmitting a signal is within the LoS, the antenna array can determine what direction the signal is coming from. The measurements in AOA consist of the direction (angle) of the received signal that is corrupted by some errors.

In practice, the beam pattern of adaptive antenna technologies is not a straight line, but rather a narrow beam and the intersection of the two narrow beams is not a single point but an area shown in Figure-6. This causes some inaccuracies in the AOA positioning. It can be seen in Figure-6 that the AOA positioning inaccuracy increases when the mobile device gets further away from the positioning receivers.

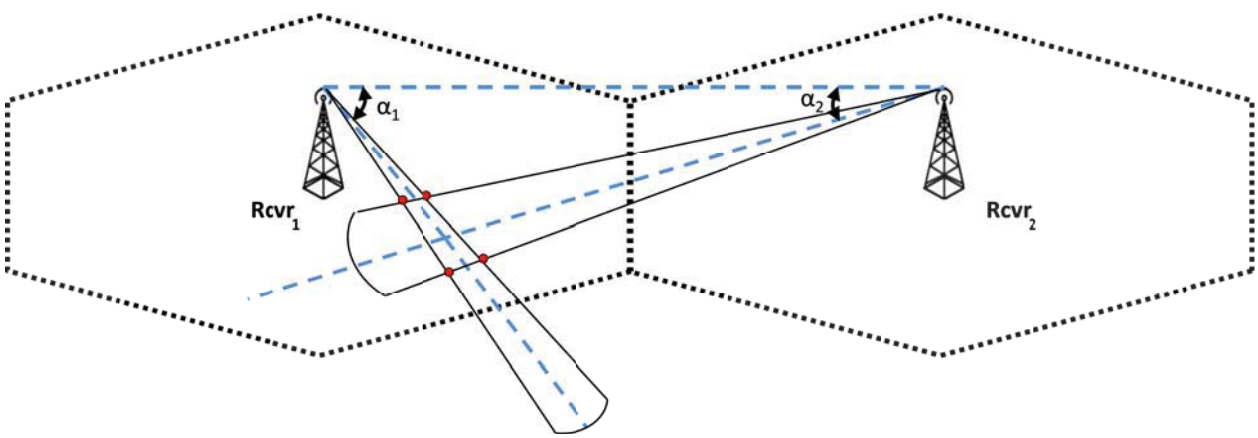


Figure-6: The intersection of the two narrow beams is the region surrounded by four dots

An advantage of AOA positioning is that it does not require either the positioning receivers or the mobile device to be synchronized. The minimum number of positioning receivers needed for the location process in AOA (i.e., two) is less than the minimum number of the positioning receivers (i.e., three) for TOA and TDOA.

On the other hand, AOA suffers from a number of disadvantages. Multipath can affect the accuracy of AOA technology; hence, AOA systems must be designed to account for multipath signals. To combat inaccuracies introduced by multipath propagation effects, more than two base stations may be employed, along with highly directional antennas. A significant drawback to an

AOA system is that it requires specialized antennas at the positioning receivers. Also, installing and aligning antenna arrays on positioning receivers can be a sensitive and costly process.

6. RF Fingerprinting Positioning

RF fingerprinting technology for positioning a mobile device utilizes a database that includes the predicted signal level along with field measurements relative to positioning receivers. Depending on the morphology, some studies suggest an error standard deviation of $\sigma_e \approx 3\ dB$ [10]. Figure-6 depicts an RF fingerprinting positioning system.

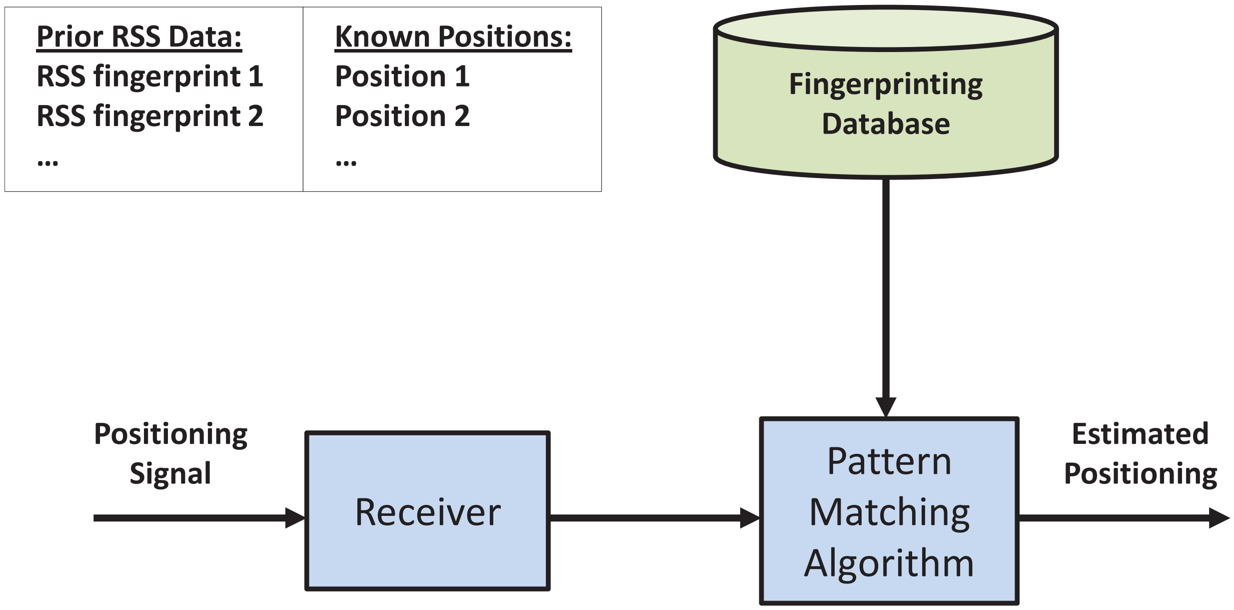


Figure-7: Fingerprinting positioning system

III. Error Models

This section summarizes the measurement accuracy levels for different positioning techniques in Table-2 [10][2].

Measurement errors for different positioning methods. P_M represents the mobile device location, P_i represents the base station "i" location, "f" represents the function as it relates the measurements to "P_M", "e" represents the measurement error with probability density function "$p_E(.)$".		
Received Signal Strength (RSS)	$y_i = K + 10\alpha \log_{10}\lvert P_M - P_i\rvert + e_i$	*4-12 dB(~90-200 meters [19])*
Time of Arrival (TOA)	$y_i = \lvert P_M - P_i\rvert + e_i$	*50-125 Meters*
Time Difference of Arrival (TDOA)	$y_i = \lvert P_M - P_i\rvert - \lvert P_M - P_j\rvert + e_i - e_j$	*10-60 Meters*
Angle of Arrival (AOA)	$y_i = f_{AOA}(P_M, P_i) + e_i$	*5º – 10º (~150 meters [20])*
RF Fingerprinting	$y_i = f_{MAP}(P_M) + e_i$	*RSS MAP 3 dB(~100 Meters [21],[22])*
GPS	$y_i = P_M + e_i$	*5 – 20 Meters*

Table-2: The measurement error for each one of the positioning techniques

[2] AFLT and Beacon Technologies are not considered in our analysis due to lack of available published research documents.

Based on [10], the RSS positioning method is the least accurate, followed by AOA and TDOA in network-based positioning technology. Some studies [10] use simulations to demonstrate that TDOA position accuracy, in NLoS (e.g., urban/dense urban) is about 100 meters for 67 percent of the calls. Other studies [3] use simulations to show that TDOA position accuracy is 50-120 meters for 67 percent of the calls. There are other studies [2] that use simulations to show that for a CDMA network, the TDOA position accuracy is about 125 meters for 67 percent of the calls. For consistency we have used the 50 - 125 meters for 67 percent of the calls for the accuracy of the TDOA technology.

IV. Methods to Improve TDOA Accuracy

One approach to improve the positioning accuracy is to use a hybrid of TDOA and AOA technologies. However, as the deployment of an AOA network-based positioning system is time consuming, expensive and complicated, another approach would be to utilize the TDOA technology with some minor adjustments. This section takes the second approach and presents a few methods in order to improve the accuracy of TDOA technology.

A. Increasing Power for Position Location

To control interference, wireless network standards provide power control for mobile devices as the mobile moves closer to the base station. In normal instances, as a mobile device moves closer to a base station, its transmit power is decreased while its path-loss is increased with other neighboring base stations. As a result, the mobile received signal level is weak at the neighboring base stations. Therefore, fewer base stations can be involved in the position location. As shown in [2] when $SINR = 10\ dB$[3], about 65 percent of the total cell areas have a position location error of less than 125 meters for 67 percent of the time.

One way to improve this error is to instruct the mobile device making E911 calls to transmit at a higher power so that its signal quality is increased at other base stations. Therefore, if the transmit power of a mobile device is increased, then the accuracy of the mobile positioning is improved. One challenge with using this solution is that when the E911 mobile's signal is received at the base station at a power higher than other mobiles, then it will degrade the overall performance of the uplink. The solution to the first problem could be resolved by the use of interference rejection techniques, which cancels out the powerful E911 mobile device and improves the signal reception of other users in the home cell. The mobile device increases the power gradually, based on the inner-loop power control mechanism instead of just switching to maximum power when a mobile needs to be located. The closed loop power control mechanism makes power corrections in fixed sized steps (typically 1 dB for cdma2000) and does not have the capability to make the mobile transmit at maximum power in a single step.

As documented in several studies [2], the mobile positioning accuracy improvement (in a CDMA network) for $SINR = 10\ dB$ is studied when mobile power is increased by 3, 6, 9 and 12 dB. These studies show that without any power increase about 65 percent of the calls have position location error less than 125 meters; however, with a power increase of:

- 3 dB - the position accuracy is improved to 72 percent of the calls,
- 6 dB - the position accuracy is improved to 81 percent of the calls,
- 9 dB - the position accuracy is improved to 87 percent of the calls, and
- 12 dB - the position accuracy is improved to 89 percent of the calls.

A conclusion can be made that it is possible to extend the position location coverage by increasing the mobile device transmit power.

[3]- The SINR includes the processing gain (see Appendix C)

B. Effect of Additional TDOA Receivers in Positioning Accuracy

Another method to improve mobile positioning accuracy is to increase the number of TDOA receivers (or location sensors) in the position estimation process. The increase in the number of TDOA receivers (in the position estimation accuracy) is studied in [2] for a CDMA network through simulations. The effect of having additional TDOA receivers (in increasing the position accuracy) is evaluated in [2] for $SINR$s of $5dB, 10\ dB, 15\ dB$. The result of the study [2] is summarized in Table -3.

SINR (dB)[4]	Percentage of Calls for 3 TDOA Receivers	Percentage of Calls for 4 TDOA Receivers
5	51%	54%
10	66%	72%
15	70%	80%

Table-3: Position accuracy for 125 meter error

As shown above, the position accuracy of the TDOA is improved when the number of TDOA receivers is increased from three (3) to four (4).

V. Improving the Location Accuracy by a Combination of an Increase in Mobile Power and Quantity of TOA Receivers:

Based on Section III, the position accuracy range for TDOA for urban areas is between 50 to 125 meters for 67 percent of the calls. Moreover, for SINR = 10 dB, the position accuracy improves by increasing: (a) the mobile transmit power and (b) the number of TDOA receivers. The effect of the increase in the mobile power is summarized in Table 4.

Mobile Power Increase (dB)	Position Accuracy (meter)	Percentage of Calls
0	50 to 125	67%
3	50 to 125	72%
6	50 to 125	81%
9	50 to 125	87%
12	50 to 125	89%

Table-4: Effect of mobile power increase position accuracy

Equation-6 can be used to determine the position accuracy for 67 percent of the calls. If the position accuracy of Δ_1 is for p_1% of mobiles and the position accuracy of Δ_2 is for p_2% of mobiles, the following relationship holds (See Appendix A):

$$\frac{\Delta_1^2}{\Delta_2^2} = \frac{\ln\left(\frac{1}{1-p_1}\right)}{\ln\left(\frac{1}{1-p_2}\right)} \quad \textbf{(Equation-11).}$$

The position accuracy for 67 percent of calls is shown in Table 5.

[4] - The SINR includes the processing gain (see Appendix C)

Mobile Power Increase (dB)	Position Accuracy (meter)	Percentage of Calls
0	50 to 125	67%
3	47 to 116	67%
6	41 to 102	67%
9	37 to 92	67%
12	35 to 88	67%

Table-5: Effect of mobile power increase position accuracy for 67 percent of calls

As described in the previous section, an additional TDOA receiver will increase the percentage of calls by 6 percent (for $SINR = 10\ dB$).

Mobile Power Increase (dB)	Position Accuracy (meter)	Percentage of Calls
0	50 to 125	73%
3	50 to 125	78%
6	50 to 125	87%
9	50 to 125	93%
12	50 to 125	95%

Table-6: Effect of mobile power increase and additional TDOA receiver in position accuracy

Equation-11 can be used to determine the position accuracy for 67 percent of the calls with an additional TDOA receiver.

Mobile Power Increase (dB)	Position Accuracy (meter)	% of Calls
0	46 to 115	67%
3	42 to 105	67%
6	37 to 92	67%
9	33 to 81	67%
12	31 to 76	67%

Table-7: Effect of mobile power increase and additional TDOA receiver in position accuracy for 67 percent of calls

Therefore an increase in mobile power along with increasing the TDOA receivers will clearly improve the position accuracy. Indeed, with these modifications, wireless network operators using TDOA technology should be able to meet the FCC's proposed indoor location accuracy standards.

VI. Conclusion

This White Paper briefly reviews the available network-based positioning technologies that can be used in wireless networks. The paper covered the pros and cons of each network-based technology. Moreover, it covered the estimated accuracy of each network-based technology. Due to a lack of available published studies concerning the AFLT and Beacon technologies, the accuracy of these technologies was not discussed.

It would be ideal to consider a hybrid solution to efficiently provide the mobile location during E911 calls to the PSAPs. While A-GPS could be used for highways and rural areas, another technology should be used to meet the FCC accuracy requirement for horizontal indoor accuracy for urban/dense urban areas. Based on the available test data, TDOA network-based technology is the available technology that provides the most accurate indoor location for a mobile handset (during an E911 call) in those morphologies where there are a sufficient number of TDOA receivers to get an accurate location. That is, in a typical urban, dense urban and suburban setting TDOA will provide the most accurate indoor location readings of any available technology.

Using the TDOA technology, this paper analyzed ways that the location of the mobile device could be improved. The TDOA mobile location accuracy can be improved by:

1) Increasing the power of the mobile device, and
2) Increasing the number of TDOA receivers.

We further estimated the accuracy of the TDOA network-based technology using nominal network performance values when the mobile power and the number of the TDOA receivers are increased. We determined that an increase in power and the number of TDOA receivers would improve the accuracy of the location of the mobile device in urban/dense urban areas. Our study revealed that using nominal network performance parameters (i.e., SINR = 10 dB), the TDOA network-based technology could improve the accuracy of the mobile location to 31 to 76 meters in 67 percent of the calls. It is conceivable that the accuracy of the TDOA network-based technology could be further improved should the network operate at: (a) a higher SINR, (b) a higher mobile power and (c) an increased number of TDOA receivers. With those simple modifications, wireless network operators using TDOA technology should be able to meet the FCC's proposed indoor location accuracy standards.

The adjustment in mobile power is already an available part of the WCDMA standards, where the network uses certain algorithms to increase or decrease mobile handset power. For instance, the mobile power adjustment is currently used by the network to control intra-network interference. Because this adjustment in the mobile handset power is already part of the industry-wide standards, allowing the mobile handset to operate at a higher power: (a) during caller locating time (which could take less than 2 seconds with TDOA technology) and (b) exclusively for E911 calls, should be easily achievable with a software patch at the network level (not the handset). Infrastructure vendors commonly release software patches for their networks (typically once a year). Since this software patch to allow a limited form of increased power for mobile handsets during 911 calls is installed at the network, it will not require any modifications to any of the handsets in service and will not require handsets to undergo any software upgrades. In short, the FCC could ask wireless carriers to make these modifications to their networks, and it could be accomplished at no cost to consumers and no cost to the wireless carriers.

Moreover, the implementation of the TDOA for indoor mobile location should be relatively easy and less costly than other currently-available location technologies since TDOA technology has already been deployed by carriers to meet the E911 Phase 2 requirements. The deployment of TDOA technology to improve indoor location accuracy will merely involve an incremental increase in the number of TDOA receivers deployed throughout the U.S., and possibly updating the TDOA receivers in a particular wireless service provider's network. Therefore, choosing the TDOA technology for the indoor mobile accuracy should be relatively easy to implement within the two year time frame recommended by the FCC.

In addition, since TDOA is technology agnostic, it can be used in 2G, 3G and 4G networks and cover all of the frequency bands. Therefore, an investment in a hybrid solution using A-GPS and the TDOA technology for mobile location purposes would be ideal as it can meet the wireless service provider's indoor and outdoor requirements using any technologies in their networks.

References

[1] Indoor Location Test Bed Report to CSRIC III-Working Group 3, E9-1-1 Location Accuracy, March 14, 2013.

[2] M. Aatique, "Evaluation of TDOA Techniques for Position Location in CDMA Systems," *Virginia Polytechnic Institute and State University,* Master of Science dissertation, September 1997.

[3] G. Sun, J. Chen, W. Guo, and K.J. Ray Liu, "Signal Processing Techniques in Network-Aided Positioning: A survey of state-of-the-art positioning designs," *IEEE Signal Processing Magazine,* pp. 12 – 23, July 2005.

[4] I. K. Adusei, K. Kyamakya, and K. Jobmann, "Mobile Positioning Technologies in Cellular Networks: An Evaluation of Their Performance Metrics," *IEEE MilComm*, pp. 1239–1244, , October 2002.

[5] Urban and Indoor Positioning Services. NextNav, LLC, October 24, 2012.

[6] H. Liu, H. Darabi, P. Banerjee, and J. Liu, "Survey of Wireless Indoor Positioning Techniques and Systems," *IEEE TRANSACTIONS ON SYSTEMS, MAN, AND CYBERNETICS - PART C: APPLICATIONS AND REVIEWS*, vol. 37, NO. 6, pp. 1067-1080, NOVEMBER 2007

[7] S. Gezici, Z. Tian, G.B. Giannakis, H. Kobayashi, A.F. Molish, H.V. Poor, and Z. Sahinglu, "Localization via ultra-wideband radios," *IEEE Signal Processing Magazine,* vol. 22, no. 4, pp. 70–84, July 2005.

[8] S.M. Kay, "Fundamentals of Signal Processing - Estimation Theory." *Englewood Cliffs*, NJ: Prentice Hall, 1993.

[9] E.L. Lehmann, Theory of Point Estimation (Statistical/Probability Series). Belmont, CA: Wadsworth & Brooks/Cole, 1991.

[10] F. Gustafsson and F. Gunnarsson, "Mobile Positioning Using Wireless Networks: Possibilities and fundamental limitations based on available wireless network measurements," *IEEE Signal Processing Magazine,* pp. 41 – 53, July 2005.

[11] Y. Okumura, E. Ohmori, T. Kawano, and K. Fukuda, "Field strength and its variability in VHF and UHF land-mobile radio service," *Rev. Elec. Commun. Lab.,* vol. 16, pp. 9–10, 1968.

[12] M. Hata, "Empirical formula for propagation loss in land mobile radio services," *IEEE Transactions on Vehicular Technology,* vol. 29, no. 3, pp. 317–325 1980.

[13] A. H. Sayed, A. R. Tarighat, and N. Khajehnouri, "Network-Based Wireless Location: Challenges faced in developing techniques for accurate wireless location information," *IEEE Signal Processing Magazine,* pp. 24 – 40, July 2005.

[14] R. Kaune, "Accuracy Studies for TDOA and TOA Localization," *IEEE Information Fusion (FUSION),* pp. 408 – 415, July 2012.

[15] M. Wylie and J. Holtzman, "The Non-Line of Sight Problem in Mobile Location Estimation," *IEEE Universal Personal Communication,* vol. 2, pp. 827 – 831, September 29 – October 2 1996.

[16] F. Gustafsson and F. Gunnarsson, "Positioning Using Time Difference of Arrival Measurements," IEEE Acoustics, Speech, and Signal Processing, vol. 6, pp. 553-556, April 2003.

[17] S. He, X. Dong, and W. S. Lu, "Asynchronous Time Difference of Arrival Positioning System," IEEE Acoustics, Speech and Signal Processing, pp. 5312 – 5315, March 31 - April 2008.

[18] Hybrid Positioning in CDMA Networks. QualComm CDMA Technologies, June 17, 2008.

[19] S. S. Saab, "An RSS Based Localization Algorithm in Cellular Networks," *ACEEE Int. J. on Network Security*, Vol. 02, No. 04, Oct 2011.

[20] P. Brida, "Location Technologies for GSM," *http://kt.utc.sk/~brida/oficialna/doc/Location%20technologies%20for%20GSM.pdf*.

[21] N. Swangmuang, P. Krishnamurthy, "Location Fingerprint Analyses Toward Efficient Indoor Positioning," *Sixth Annual IEEE International Conference on Pervasive Computing and Communications,*

[22] Prashant Krishnamurthy, "Position Location Using Radio Fingerprints in Wireless Networks," *http://www.pitt.edu/~dtipper/2011/Slides1_09.pdf*.

[23] D.N. Hatfield, "A report on technical and operational issues impacting the provision of wireless enhanced 911 services," *Federal Communications Commission, Tech. Rep.,* 2002.

[24] A. Roxin, J. Gaber, M. Wack, A. Nait-Sidi-Moh, "Survey of Wireless Geolocation Techniques," *IEEE Globecom Workshops,* Washington, DC, November 2007

[25] T. Tonteri, "A Statistical Modeling Approach to Location Estimation," *University of Helsinki,* Master of Science dissertation, May 2001.

[26] E. A. Prigge , "A Positioning System With No Line-Of-Sight Restrictions For Cluttered Environments," *Stanford University*, Ph.D. dissertation, August 2004.

[27] K.J. Krizman, T.E. Biedka, and T.S. Rappaport, "Wireless position location: Fundamentals, implementation strategies, and sources of error," *Proc. IEEE Vehicular Technology Conf.,* vol. 2, pp. 919–923, June 1997.

[28] G. A. Mizusawa, "Performance of Hyperbolic Position Location Techniques for CDMA," *Virginia Polytechnic Institute and State University,* Master of Science dissertation, August 1996.

[29] Y. Zhao, "Standardization of mobile phone positioning for 3G systems," *IEEE Communications Magazine,* vol. 40, no. 7, pp. 108–116, July 2002.

[30] J. J. Caffery, Jr., G. L. Stuber, "Overview of Radiolocation in CDMA Cellular Systems," *IEEE Communications Magazine*, pp. 38-45, April 1998.

[31] C. Drane, M. Macnaughtan, and C. Scott. "Positioning GSM telephones," *IEEE Communications Magazine,* vol. 36, no. 4, pp. 46–54, Apr. 1998.

[32] G. Wölfle, R. Hoppe, D. Zimmermann, F. M. Landstorfer, "Enhanced Localization Technique within Urban and Indoor Environments based on Accurate and Fast Propagation Models," *http://docenti.ing.unipi.it/ew2002/proceedings/179.pdf*.

[33] S. Venkatraman, J.Caffery, Jr., and H. R. You, "A Novel ToA Location Algorithm Using LoS Range Estimation for NLoS Environments," *IEEE Transactions On Vehicular Technology,* vol. 53, No. 5, pp. 1515 – 1524, September 2004.

[34] I. K Adusei, K. Kyamakya, K. Jobmann, "Mobile Positioning Technologies in Cellular Networks: An Evaluation of their Performance Metrics," *IEEE MILCOMM,* vol 2, pp. 1239-1244, October 2002.

[35] S. Tekinay, E. Chao, R. Richton, "Performance Benchmarking for Wireless Location Systems, *IEEE Communications Magazine,* pp. 72 – 76, April 1998.

[36] G.G. Raleigh and T. Boros, "Joint space-time parameter estimation for wireless communication channels," *IEEE Trans. Signal Processing*, vol. 46, pp. 1333–1343, May 1998.

[37] K. Pahlavan, X. Li, J. P. Mäkelä, "Indoor Geolocation Science and Technology," *IEEE Communications Magazine*, pp. 112-118, February 2002.

[38] H. Hashemi, "Pulse Ranging Radiolocation Technique And its Application to Channel Assignment in Digital Cellular Radio," *IEEE Vehicular Technology Conference,* pp. 675 – 680, May 1991.

[39] Y.Qi and H. Kobayashi, "Cramer-Rao lower bound for geolocation in non-line-of-sight environment," in Proc. *IEEE Acoustics, Speech and Signal Processing*, pp. 2473–2476, Orlando, FL, May 2002.

Appendix A

First, let us define circular error probability (CEP) that is used to evaluate the accuracy of positioning technologies.

The Circular Error Probability (CEP) is the radius of a circle, centered about the actual position, that for a specific percentage (e.g., 50 percent) of callers where the error is lower. For example, if CEP67 is 125 meters, then the positioning accuracy is lower than 125 meters for 67 percent of the calls.

Now, let us assume that the accuracy is 125 meters for a different percentage of the calls (e.g., 80 percent or CEP80). The question is what is the CEP67? In other words, what is the positioning accuracy for 67 percent of the calls?

Without a loss of generality, let us assume that the actual position of the E911 caller is $(0,0)$. Now let us assume that the estimated position of the E911 caller is (X, Y), where X and Y are the errors along the $X-$axis and axis and $Y-$axis, respectively. Let us assume that:

- $X \sim N(0,\sigma), i.e., p_X = 1/\sqrt{2\pi\sigma}\, e^{-X^2/2\sigma^2}$
- $Y \sim N(0,\sigma)\ i.e., p_Y = 1/\sqrt{2\pi\sigma}\, e^{-Y^2/2\sigma^2}$
- X and Y are identically and independent random variables

The third assumption is not quite accurate and just simplifies the problem. Let us define the random variable u as below:

$$u = r^2 = X^2 + Y^2$$

It can be easily shown that the random variable u is an exponential random variable with a of mean $2\sigma^2$:

$u \sim EXP(2\sigma^2), i.e., p_r = \frac{1}{2\sigma^2} e^{-u/2\sigma^2}$ for $u \geq 0$.

Let us assume that the position accuracy error is Δ_1 for p_1 percent of the calls, or the probability that r is lower than a given value of Δ_1 is equal to p_1:

$$p_1 = P[r \leq r_1] = \int_0^{\Delta_1^2} \frac{1}{2\sigma^2} e^{-u/2\sigma^2} du = 1 - e^{-\Delta_1^2/2\sigma^2}$$

In other words,

$$\Delta_1^2 = 2\sigma^2 \ln \frac{1}{1-p_1}$$

If the probability that the position accuracy error is Δ_2 for p_2 percent of the calls, then:

$$\Delta_2^2 = 2\sigma^2 \ln \frac{1}{1-p_2}$$

By combining the last two equations, we obtain:

$$\frac{\Delta_1^2}{\Delta_2^2} = \frac{\ln \frac{1}{1-p_1}}{\ln \frac{1}{1-p_2}}$$

Appendix B

Case Study to Estimate the Number of TOA Receivers

The most efficient approach to provide an accurate location of E911 calls is the deployment of a hybrid A-GPS and TDOA technology. To improve the accuracy of indoor E911 calls, it will be important to deploy an adequate number of UTDOA stations in urban areas. Urban areas based on the 2010 population have been identified by the Census Bureau (http://www.census.gov/cgi-bin/geo/shapefiles2010/main). Using the Census Bureau's definition, it is possible to calculate the total area and population for each of the defined urban areas. The list of urban areas, along with the calculated areas and total population, is included as Exhibit B-1. The urban areas are located in the United States as depicted in Figure B-1.

Based on Census 2010, total urban areas encompass 110,708.70 (please see Exhibit B-1) square miles and cover a total population of 241,206,854 (the population figure is based on Census 2000).

To estimate the number of UTDOA stations in an urban area, it is important to first estimate the cell radii of urban cell sites. The cell radii for different cell morphology for a1900 MHz network with SINR = 10 dB is included in Exhibit B-2. The link budget estimates the cell radius for an urban cell to be 0.75 miles (1.2 Km).

The total estimated number of UTDOA stations to cover the urban areas as defined by the Census 2010, is summarized in Table B-1. The summary uses the cell radius for urban areas of 0.75 miles and the estimation technique shown in Exhibit B-3.

Total Urban Area (in 50 States + PR)	Estimated Urban Cell Radius (Mile)	No. of UTDOA stations Per Sq. Mile	Total Number of Positioning Sensors (Assuming 1:1 Ratio)
110,708.70	0.75	0.59	65,563

Table B-1 Estimated Number of Positioning Receiver for Urban Area

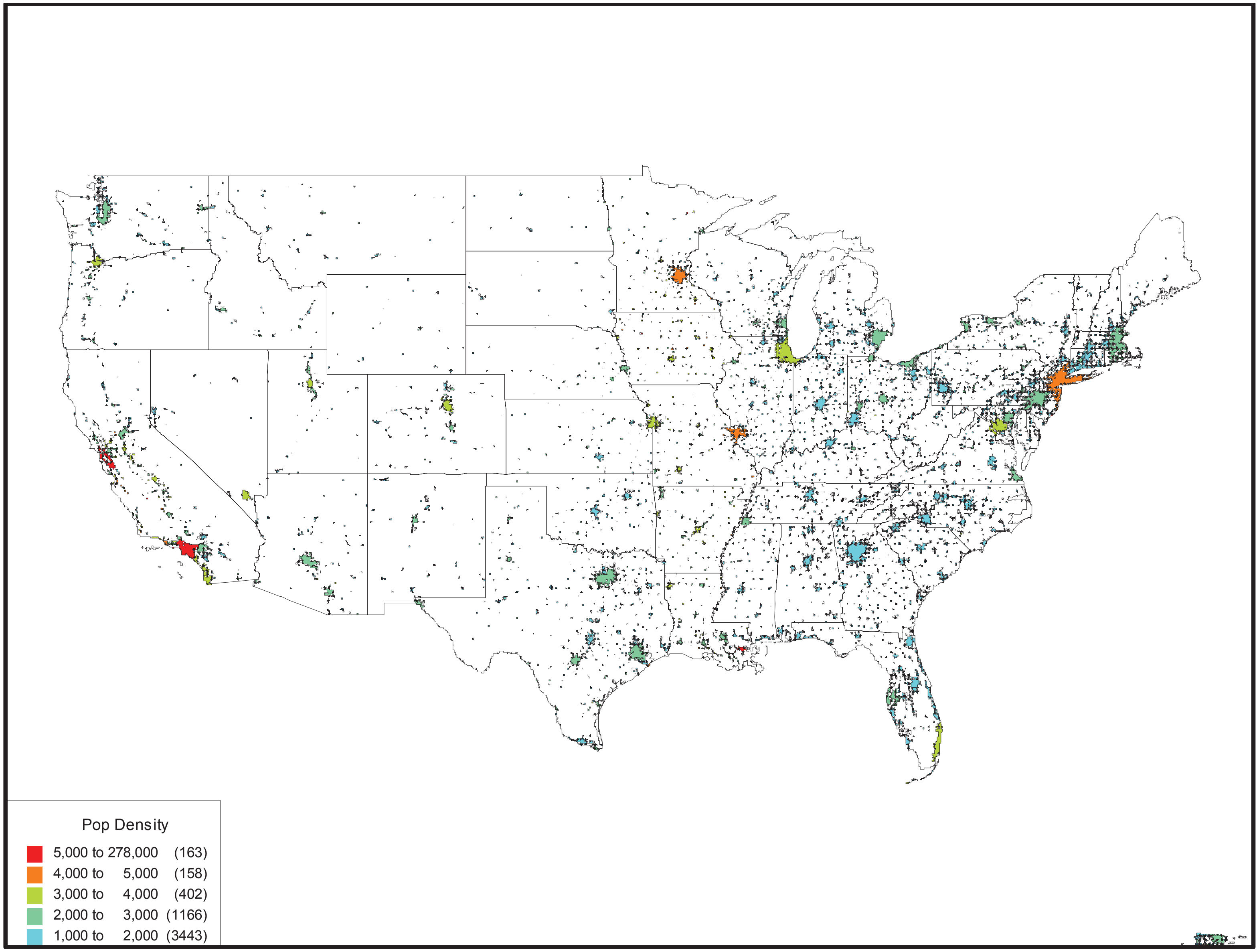


Figure B-1 – Urban Area for the US as Defined by Census 2010

Exhibit B-1

Urban Areas As Defined by Census 2010

Exhibit B-1.pdf

Exhibit B-2

Link Budget

Link Budget

Composite Probability Of Service & Required Fade Margin

Environment Type ("morphology")	Building Median Loss dB	Building Std. Dev, dB	Outdoor Std. Dev dB	Composite Standard Deviation	Desired Reliability at Cell Edge, %	Fade Margin dB	Path loss Exponent	Area Reliability %
Urban	15	9	8	12.04	75.0%	8.12	4	88.1%

	Fraction of Area FU	Fade Margin	Mean Signal Xbar	Rx Threshold Xo	Std Deviation Sigma	Cell Edge Probability Pxo	e (constant)	Prop Loss Coeffic. N	Calc Const. A	Calc Const. B
Urban	88.1%	8.122	-105.58	-113.7	12.04	75.0%	2.718	4.0	-0.477	1.020

Link Budget

Reverse Link Budget

Term or Factor	Given		Urban
MS TX Power (dbm) (+)	25.14		
MS Antenna Gain and Body Loss (+/-)	0		
MS EIRP (dBm) (+)			25.14
Fade Margin, (dB) (-)			-8.12
Soft Handoff Gain (dB) (+)			4
Ambient Noise (-)			-2
Receiver Interf. Margin (dB) (-)			-3
Building Penetration Loss (dB) (-)			-15.00
BTS RX antenna Gain (dBi) (+)			17
BTS cable loss (dB) (-)			-3
kTB	-113.1		
BTS Noise Figure (dB)	6.5		
Eb/Nt (dB)	10		
Processing Gain	21.1		
BTS RX Sensitivity (dBm) (-)			**-117.7**
Survivable Uplink Path Loss (dB) (+)			**132.7**

Forward Link Budget

Term or Factor	Given		Urban
BTS TX Power (dBm) (+)			**48**
BTS TX Power (watts)			**63.10**
% Power for Traffic Channels			**74.0%**
Number of Traffic Channels in Use			**19**
BTS Cable Loss (dB) (-)			**-3**
BTS TX Antenna Gain (dBi) (+)			**17**
BTS EIRP/Traffic Channel (dBm) (+,-)			**47.9**
Fade Margin (dB) (-)			**-8.12**
Receiver Interference Margin (db) (-)			**-3**
Ambient Noise (-)			**-2**
Building Penetration Loss (dB) (-)			**-15.0**
MS Antenna Gain & Body Loss (dB) (+,-)			**0**
kTB (dBm/14.4 KHz.)	-113.1		
Subscriber RX Noise Figure (dB)	10.5		
Eb/Nt (dB)	10		
Processing Gain	21.1		
Subscriber RX Sensitivity (dBm) (-)			**-113.7**
Survivable Downlink Path Loss, dB (+)			**133.5**

Explore propagation model to figure coverage radius of cell.

Frequency, MHz.	1900
Subscriber Antenna Height, M	1.5
	Urban
Base Station Antenna Height, M	30

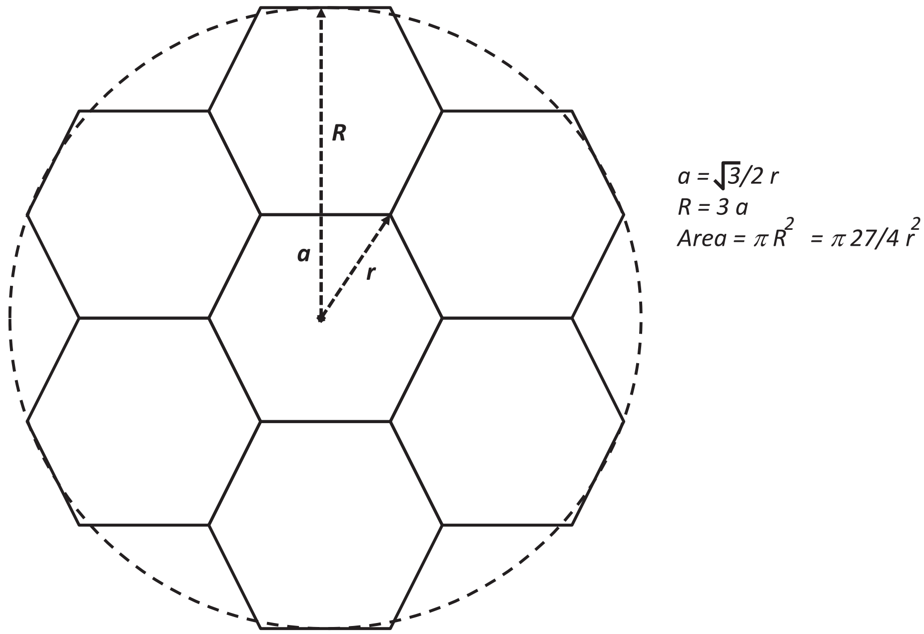
	Urban
Environmental Correction, dB	-5
Coverage Radius, kM	1.20
Coverage Radius, Miles	0.75

#of UTDOA Stations/Sq. Miles	0.592

Exhibit B-3

Methodology Used to Estimate the Number of UDTOA Stations

The radius estimated from the link-budget is denoted by "r" in mile. The area of the seven hexagons is approximated by the area of the circle around the seven hexagons shown in the figure below:



The area covered by seven sensors is the approximated by the area of the circle.

Clearly,

$$a = \frac{\sqrt{3}}{2} r, \qquad R = 3 \times a$$

The area covered by the seven sensors in the figure is approximately as follows:

$$Area \approx \pi R^2 = \pi \frac{27}{4} r^2$$

The number of sensors needed per square mile is:

$$Avg\ \#\ of\ sensors\ per\ square\ mile = \frac{7}{Area} = \frac{28}{\pi \times 27 r^2} = \frac{0.33}{r^2}$$

Appendix C

System Processing Gain

The $SINR$ of 10 dB, used in this report includes the processing gain. The processing gain for the cdma and UMTS systems are shown below.

Let us assume that the system sampling rate is 10 $Ksps$(K samples per second). The chip rates of UMTS and cdma are as follows:

UMTS
Chip Rate = 3.84 Mcps
Processing Gain = $10 \times \log_{10}\left(\frac{3.84\times10^6}{10\times10^3}\right) \approx 25.8\ dB$

cdma
Chip Rate = 1.2288 Mcps
Processing Gain = $10 \times \log_{10}\left(\frac{1.2288\times10^6}{10\times10^3}\right) \approx 21\ dB$

Therefore, for $SINR = 10\ dB$, the actual required $SINR$ in UMTS and cdma are $\approx -16\ dB\ and - 11\ dB$ respectively.

Author's Biography

Mehdi Alasti, Ph.D.

Mehdi Alasti received his PhD in the area of Wireless Communication Networks from University of Maryland at College Park in Jan 2001, with the focus on MAC Layer & Application Layer cross layer design for Integrated Voice & Data Wireless Networks. Afterwards, Dr. Alasti was one of the founders of Zagros Networks Inc, a fables semiconductor start-up company headquartered in Rockville, MD (2001 - 2003), where he worked as a Principle Engineer reporting to the VP of Engineering. From July 2003 - December 2003 Dr. Alasti joined the Electrical & Computer Engineering Department of University of Maryland as a Faculty Research Associate. From January 2004 – December 2006, Dr. Alasti joined Advanced Technology Group of Airvana Inc (www.airvana.com Chelmsford, MA) reporting to Airvana CTO and working on the next generation wireless technologies and next generation products for Airvana Inc. Dr. Alasti joined AdGen Telecom Group in 2007 and is working as a Technology Strategist consultant focusing in 4G (WiMAX) and Core Network, LTE RAN, EPC Core (security & authentication, QoS, CSFB, SRVCC, VoLTE, E911, roaming/IPX, data offloading), IMS, LTE RAN, cloud based OSS/BSS, and municipal WiFi network. Dr. Alasti has published a number of technical papers in IEEE and holds a number of patents.